Mechanical Properties of Solids

1. A composite wire of uniform diameter 3.0 mm consisting of a copper wire of length 2.2 m and a steel wire of length 1.6 m stretches under a load by 0.7 mm. Calculate the load, (in N) given that the Young's modulus for copper is 1.1×10^{11} Pa and for steel is 2.0×10^{11} Pa.
2. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that g = 3.1 $\pi \mathrm{ms}^{-2}$, what will be the tensile stress (in Nm^{-2}) that would be developed in the wire?
3. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus (in Nm^{-2}) of rubber is closest to :
4. Young's moduli of two wires A and B are in the ratio 7:4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R (in mm) is close to :
5. In an environment, brass and steel wires of length 1 m each with areas of cross section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress (in Nm^{-2}) required to produce a net elongation of 0.2 mm is, [Given, the Young's modulus for steel and brass are, respectively, $120 \times 10^9\ \mathrm{N/m^2}$ and $60 \times 10^9\ \mathrm{N/m^2}$]
6. The elastic limit of brass is 379 MPa. What should be the minimum diameter (in mm) of a brass rod if it is to support a 400 N load without exceeding its elastic limit?
7. A 2 m long rod of radius 1 cm which is fixed from one end is given a twist of 0.8 radians. The shear strain developed will be
8. The compressibility of water is 4×10^{-5} per unit atmospheric pressure. The decrease in volume (in cm^3) of 100 cm^3 of water under a pressure of 100 atmosphere will be
9. A uniform cube is subjected to volume compression. If each side is decreased by 1%, then bulk strain is
10. What is the bulk modulus (in Pa) of water for the given data: Initial volume = 100 litre, pressure increase = 100 atmosphere, final volume = 100.5 litre (1 atmosphere = 1.013×10^5 Pa)
11. The breaking stress of the material of a wire is $6 \times 10^6 \mathrm{Nm}^{-2}$. Then density ρ of the material is $3 \times 10^3\ \mathrm{kg\ m}^{-3}$. If the wire is to break under its own weight, the length (in metre) of the wire made of that material should be (take $g = 10\ \mathrm{ms}^{-2}$)
12. When a certain weight is suspended to a long uniform wire its length increases by one cm. If the same weight is suspended to another wire of the same material and length but having a diameter half of the first one, the increase in its length (in cm) will be
13. A wire of length 50 cm and cross-sectional area of 1 mm^2 is extended by 1 mm. The required work (in joule) will be ($Y = 2 \times 10^{10} \mathrm{Nm}^{-2}$)
14. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2 m long. The wires at each end are of copper and middle one is of iron. The ratio of the diameters of the copper to iron wire if each has the same tension is [Young's modulus of elasticity for copper and iron are $110 \times 10^9\ \mathrm{N/m^2}$ and $190 \times 10^9\ \mathrm{N/m^2}$ respectively.]
15. The Marina Trench is located in the Pacific Ocean and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. The change in the volume (in m^3) of the ball when it reaches to the bottom is [Bulk modulus for steel = $1.6 \times 10^{11}\ \mathrm{N/m^2}$ J

SOLUTIONS

1. **(176.8)** The extension $\Delta L_{copper} = \frac{F\ell_1}{AY_c}$

$$= \frac{F \times 2.2}{\pi(0.003)^2 \times 1.1 \times 10^{11}}$$

and $\Delta L_{steel} = \frac{F\ell_2}{AY_s}$

$$= \frac{F \times 1.6}{\pi(0.003)^2 \times 2 \times 10^{11}}$$

Given $\Delta L_{copper} + \Delta L_{steel} = 0.7 \times 10^{-3}$

After simplifying above equations, we get

$$F = 176.8\,\text{N}$$

2. **(3.1×10^6)** Given,

Radius of wire, r = 2 mm

Mass of the load m = 4 kg

Stress $= \frac{F}{A} = \frac{mg}{\pi(r)^2}$

$$= \frac{4 \times 3.1\pi}{\pi \times (2 \times 10^{-3})^2} = 3.1 \times 10^6 \text{ N/m}^2$$

3. (2.3×10^6) When a catapult is stretched up to length ℓ, then the stored energy in it = Δk. E

$$\Rightarrow \frac{1}{2} \cdot \left(\frac{YA}{L}\right)(\Delta l)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow y = \frac{mv^2 L}{\Delta(\Delta l)^2}$$

m = 0.02 kg

v = 20 ms^{-1}

L = 0.42 m

A = $(\pi\, d^2)/(4)$

d = 6×10^{-3} m

$\Delta\ell = 0.2$ m

$$= \frac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04}$$

$= 2.3 \times 10^6 \text{ N/m}^2$

So, order is 10^6.

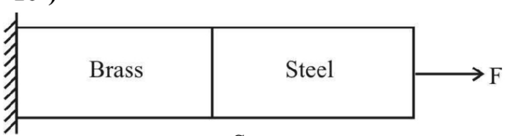
4. **(1.75)** $\Delta_1 = \Delta_2$

or $\frac{Fl_1}{\pi r_1^2 y_1} = \frac{Fl_2}{\pi r_2^2 y_2}$

or $\frac{2}{R^2 \times 7} = \frac{1.5}{2^2 \times 4}$

$\therefore R = 1.75$ mm

5. **(8×10^6)**



Young modulus, $Y = \frac{\text{Stress}}{\left(\frac{\Delta l}{L}\right)}$

Let σ be the stress

Total elongation $\Delta l_{net} = \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2}$

$\Delta l_{net} = \sigma\left[\frac{1}{Y_1} + \frac{1}{Y_2}\right]$ $\quad [\because L_1 = L_2 = 1\text{m}]$

$\sigma = \Delta l\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)$

$= 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9$

$= 8 \times 10^6 \frac{N}{m^2}$

6. **(1.15)** Stress $= \frac{F}{A} = \frac{400 \times 4}{\pi d^2} = 379 \times 10^6\ \text{N/m}^2$

$\Rightarrow d^2 = \frac{400 \times 4}{379 \times 10^6 \pi}$

$d = 1.15$ mm

7. **(0.004)** $r\theta = L\phi \Rightarrow 10^{-2} \times 0.8 = 2 \times \phi \Rightarrow \phi = 0.004$

8. **(0.4)** Compressibility $= \frac{1}{B} = \frac{\Delta V / V}{P}$. Here, P = 100 atm,

Compressibility = 4×10^{-5} and V = 100 cm^3.

Hence, $\Delta V = 0.4\ \text{cm}^3$

9. **(0.03)** If side of the cube is L then $V = L^3 \Rightarrow \frac{dV}{V} = 3\frac{dL}{L}$

$\therefore$ % change in volume = 3 × (% change in length)

= 3 × 1% = 3%

$\therefore$ Bulk strain $\frac{\Delta V}{V} = 0.03$

10. **(2.026 × 10^9)** Here,

$\Delta V = 100.5 - 100 = 0.5$ litre $= 0.5 \times 10^{-3}\ \text{m}^3$;

$P = 100$ atm $= 100 \times 1.013 \times 10^5$ Pa

$V = 100$ litre $= 100 \times 10^{-3}\text{m}^3$

Bulk modulus $= B = \frac{P}{DV/V} = \frac{PV}{\Delta V}$

$= \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$

$\Rightarrow \quad B = 2.026 \times 10^9$ Pa

11. **(200)** Breaking stress $= \frac{\text{Force}}{\text{area}}$

The breaking force will be its own weight.

$F = mg = V\rho g = \text{area} \times \ell\rho g$

Breaking stress $= 6 \times 10^6 = \frac{\text{area} \times \ell \times \rho g}{\text{area}}$

or $\quad \ell = \frac{6 \times 10^6}{3 \times 10^3 \times 10} = 200$ m.

12. **(4)** $\ell = \frac{FL}{AY} \Rightarrow \ell \propto \frac{1}{r^2}$ (F, L and Y are same)

$\frac{\ell_1}{\ell_2} = \left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{\ell_1}{\ell_2} = \left(\frac{r_2}{2r_2}\right)^2 = \frac{1}{4}$

$\Rightarrow \ell_2 = 4\ell_1 = 4$ cm

13. **(2×10^{-2})** $W=\frac{YA\ell^2}{2L}=\frac{2\times10^{10}\times10^{-6}\times(10^{-3})^2}{2\times50\times10^{-2}}=2\times10^{-2}$ J

14. **(1.31)** Each wire has same tension so each wire will have same extension. Also as they have same length, each wire will have same strain

$$Y=\frac{Fl}{A\Delta l}=\frac{Fl}{\pi(D/2)^2\Delta l}=\frac{4Fl}{\pi D^2\Delta l}$$

$$\therefore\quad D^2\propto\frac{1}{Y}$$

$$\therefore\quad \frac{D_{Cu}}{D_{iron}}=\sqrt{\frac{Y_{iron}}{Y_{Cu}}}=\sqrt{\frac{190\times10^9}{110\times10^9}}=\sqrt{\frac{19}{11}}=1.31$$

15. **(2.2×10^{-4})** From question,
$P = 1.1\times10^8$ Pa, $V = 0.32$ m^3, $B = 1.6\times10^{11}$ Pa;
Change in volume,

$$\Delta V=\frac{PV}{B}=\frac{1.1\times10^8\times0.32}{1.6\times10^{11}}=2.2\times10^{-4}\,\text{m}^3.$$